76,808 square miles of land in Nebraska, so that, assuming a uniform areal distribution and that no place would be visited a second time, more than 10,000 years would be required before all localities in the State would experience a tornado. Therefore, the chance of one encountering such a storm in the State, once in more than 10,000 years, is remote indeed. Furthermore, the average annual number of deaths from tornadoes in Nebraska during this period was approximately 1.5. Therefore, as the population of the State is around one and a half million the chance that an individual will lose his life in a tornado is only about one in a million.

A PERIODOGRAM INVESTIGATION OF SHORT-PERIOD SUNSPOT CYCLES

By DINSMORE ALTER

[Griffith Observatory, Los Angeles, Calif., December 1937]

The present report concerns an investigation, the calculations of which were made several years ago and laid aside, because at the time it seemed impossible to make any satisfactory interpretation of quite definite but nevertheless of apparently contradictory results. The remarkable distribution of sunspots during the past year has now thrown a little light upon what may have been happening during the epoch for which this investigation was made, and makes it appear best to publish the results of an extremely long calculation. The data are those gathered by Elsa Frenkel and used by her in 1913.[1] From these data Dr. Frenkel (now Dagobert) computed a Schuster periodogram in an attempt to find short-period solar variations. She did find some evidence of such, especially one of 69.4 days. Her data began with January 1877 and were continued through 1911. She published them as 5-day means of the Wolf relative numbers. On account of the weakness of the Schuster method, it was very difficult to find any definite results.

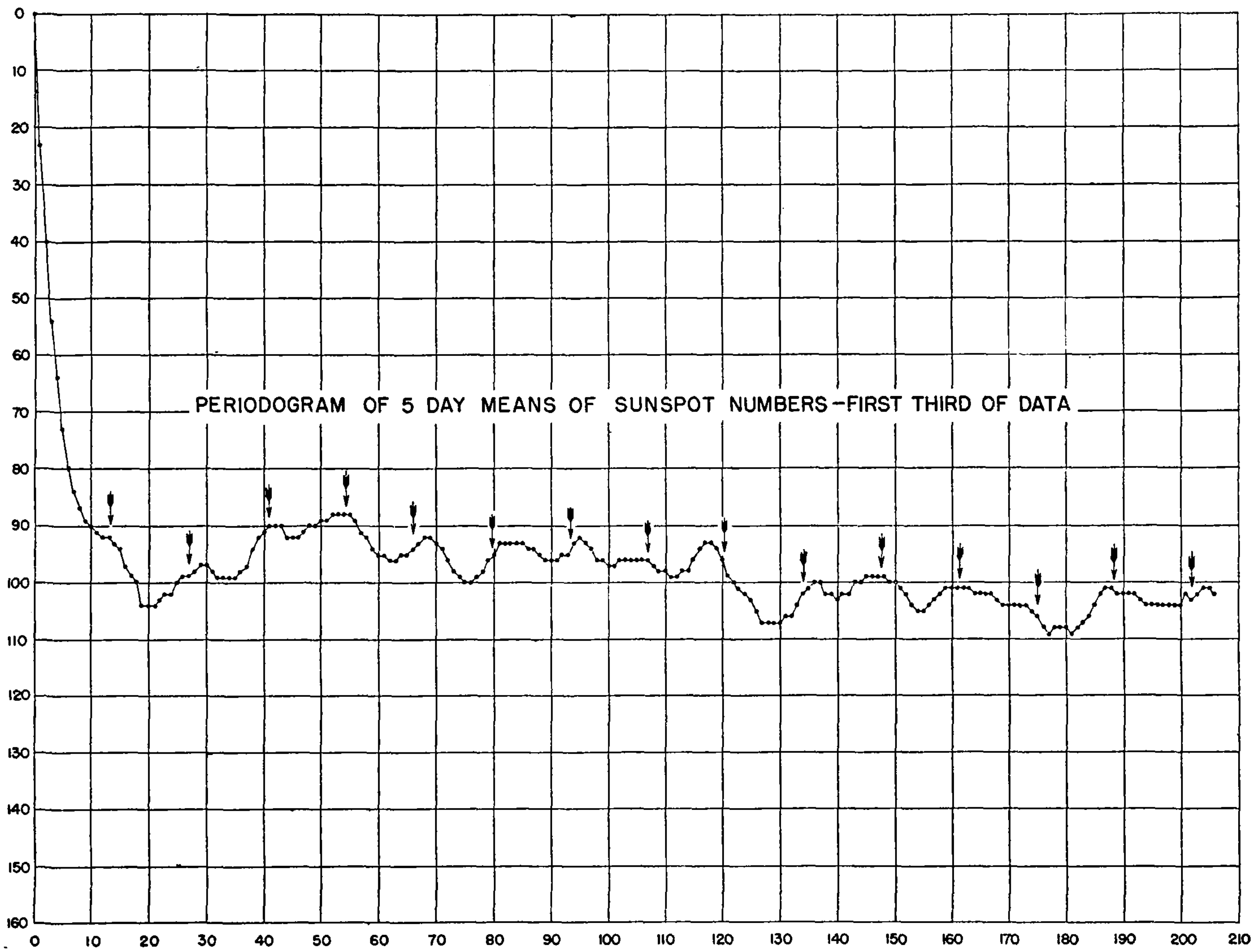

FIGURE 1.

In the present investigation a new form of periodogram of linear type was used.[2] This periodogram uses as an

[1] Frenkel, Elsa. 1913. Untersuchungen über kurzperiodische Schwankungen der Häufigkeit der sonnenflecken. Publikationen der Sternwarte des eidgenössischen Polytechnikums. Band V.

[2] Alter, Dinsmore. 1937. A Simple Form of Periodogram. The Annals of Mathematical Statistics. Vol. 7, No. 2, p. 121.

index the standard deviation of the errors of predictions by the hypothesis that data will be repeated after a given lag or cycle. As in the case of all other periodograms, these lags are tried for all integral values. The factor 0.85 has been included in the indices in the accompanying diagrams to give the probable errors of predictions by the various trial hypotheses. In all more recent calculations this factor has been changed, so that standard deviations instead of probable errors are used in them. The number of pairs of data matched for each point on the various

In any form of periodogram calculation a difficulty is always encountered when there exists in the data some long cycle or period which is not under investigation. As an example, in meteorological data, we usually eliminate the annual term. In the case of these sunspot data the troublesome term has a length of a little more than 11 years. The range of periodicities investigated here has a maximum length of not more than 2 years. The fact that the 11-year cycle varies in length and in intensity causes difficulty in eliminating it from solar data.

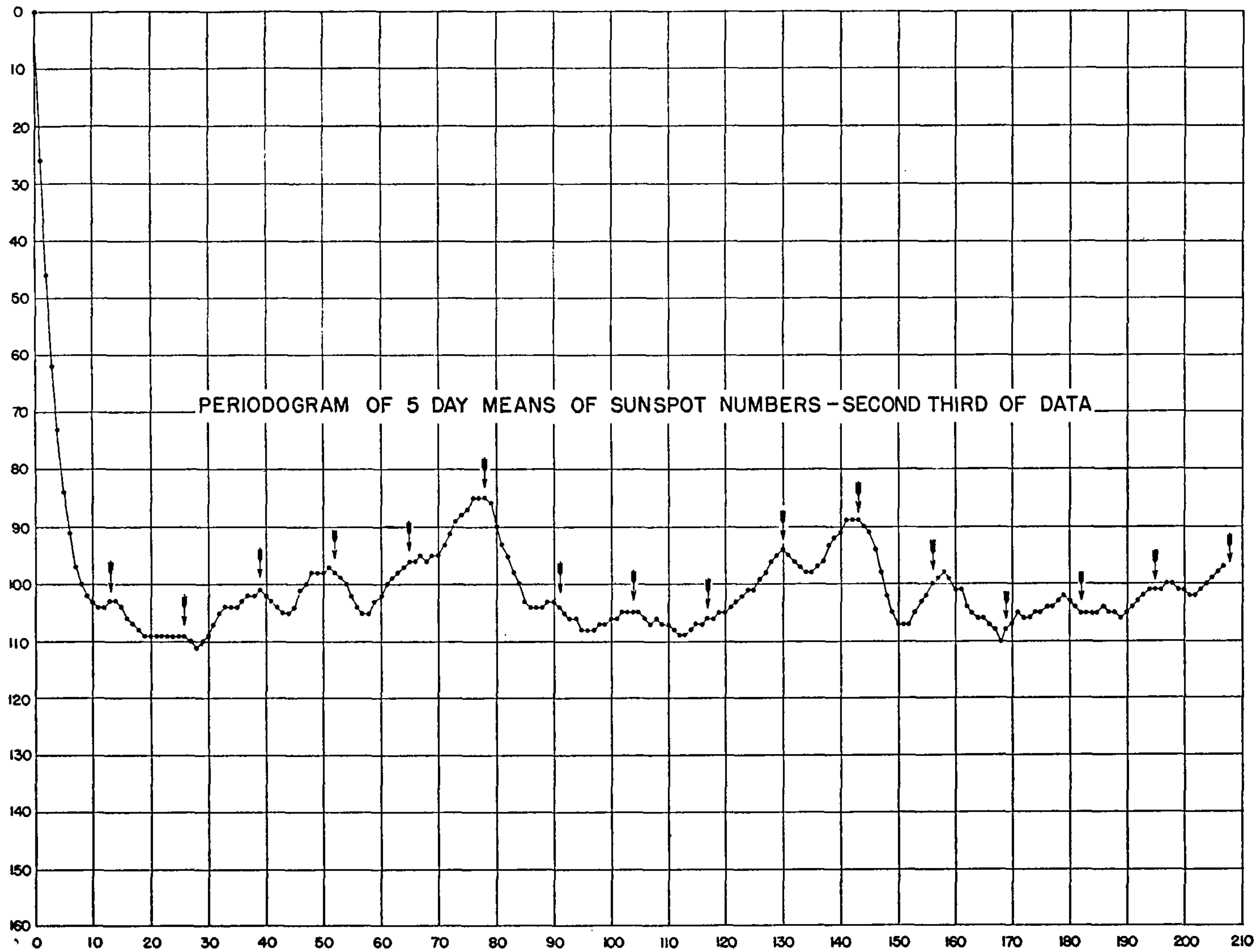

FIGURE 2

curves averages close to 750. The probable errors of these indices themselves are, therefore, extremely small, despite the fact that adjacent data are not independent of each other.

Minima of sunspots occurred at 1878.9, 1889.6, 1901.7, and 1913.7. These 35 years of data, therefore, without serious error may be considered as covering three sunspot cycles from minimum to minimum. It appeared practical to divide them approximately into thirds and to compute periodograms separately, to learn whether there is significant resemblance of short-period terms from one sunspot cycle to the next. Finally all the data were combined into one periodogram in an attempt to find out whether any cycles of solar variation had persisted through perhaps several hundred repetitions.

The following method was used: The mean was taken of each 59 consecutive values of the data; for example, the first 59 were averaged, then numbers 2 to 60, 3 to 61, and so on until the end. The curve plotted from these means must exhibit terms of length greater than 4 or 5 years quite plainly. Terms as long as 11 years are scarcely damped at all. On the other hand, any period of 295 days or of a submultiple of that value is entirely eliminated from the data. It is an easy matter of calculation to show that no term of length less than, say, 400 days would remain in these means with any appreciable amplitude. Next, each mean was subtracted from the middle datum of the 59 data used in securing it. These differences are almost entirely free from long-period terms. However, they do contain the short-period terms nearly unchanged, although with a very slight bias toward the submultiples of 295 days.

These differences were used in computing the periodogram. The numbers were in general quite small, which shortened the work. The resulting periodograms for each third of the data are exhibited as figures 1, 2, and 3. The abscissae are expressed in terms of Dr. Dagobert's pentads. For a lag of 0, the periodogram factor is, of course, zero. It has been stated earlier that the ordinates used for the periodograms are the probable errors of predictions by the various hypotheses. Each abscissa represents where the best repetition of periodogram peaks is found. These hold quite nicely for a cycle of length 68.0 days until after it has b en repeated seven times. From then on the agreement seems perhaps a little better than accidental although there is little that can be claimed for it. Such a pattern is that which would be due to a cycle of unconstant length averaging at the value stated. When such cycles exist in data, the first peaks must show up most strongly, but as repetition is continued, they must become, in general, less clear. If this 11-year sunspot cycle had been the only one investigated, the conclusion regarding the

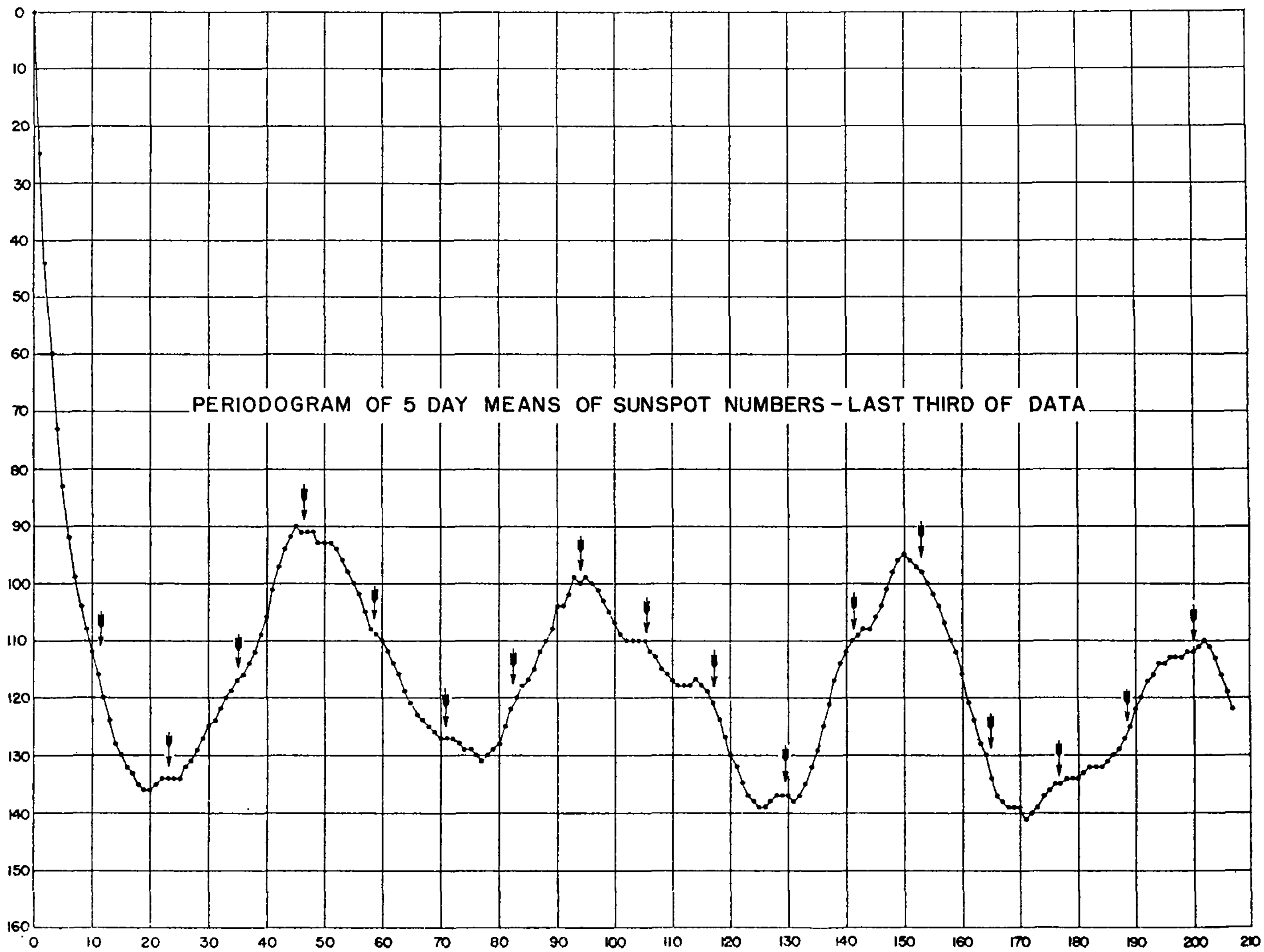

FIGURE 3.

one hypothesis concerning the existence of a cycle. Chance or, in other words, zero correlation, gives an ordinate of 8.95 for the first third of the data. The second third of the data has 9.37 for the random expectancy of this probable error. The last third has 10.94. For the first half dozen pentads the curve drops rapidly, because the dependence of one pentad on the preceding ones extends over little more than a month. After a few pentads, independence is reached and from there on the pattern is determined by cycles which exist in the data, such cycles being due either to accidental variations or to more or less permanent physical causes.

The first third of the data shows very little amplitude to the swings. Arrows have been drawn at intervals existence of short cycles certainly would have been negative. These data do not by themselves give a periodogram pattern strong enough to carry conviction. However, if in other stretches of data amplitudes are found which cannot be accidental, any *a priori* argument against the 68.0-day cycle must fail, and it must be considered as quite probably having a real existence during the years studied.

With the second third of the data, a very different type of pattern begins to appear. Two peaks stand out high above all others. One of these is spread over the lags from 75 to 79 pentads; the other from 140 to 145. The latter is, within accidental range, double the former. Counting the small peaks between, it is found that they are spaced at quite regular intervals with the sixth one occurring at the first of these high peaks and the eleventh occurring at the second The twelfth one follows almost perfectly at a sharp peak at pentad 158. The variations are too large to be accidental, when one considers that there are 800 pairs of data used in the calculation of each point. Examination of the peak between 70 and 80 shows that its left side is much less steep than the right. The same thing occurs at the second peak. The pattern is almost perfectly that which would be secured from a long cycle of length about 360 days with one of 65.0 days superposed case—65.0 against 68.0 days. Such a peculiarity of distribution of sunspots as has occurred during 1936–37, when one-half of the sun has been far more spotted than the other, could easily account for a far greater discrepancy than is found here, even if one were investigating an hypothesis of a periodicity of unchanging length.

With the last third of the data, the periodogram becomes much more striking than it was before. There has now appeared a very definite cycle of length a little less than 250 days that far outshadows everything else. Its amplitude is so large, and the regularity of the peaks obtained

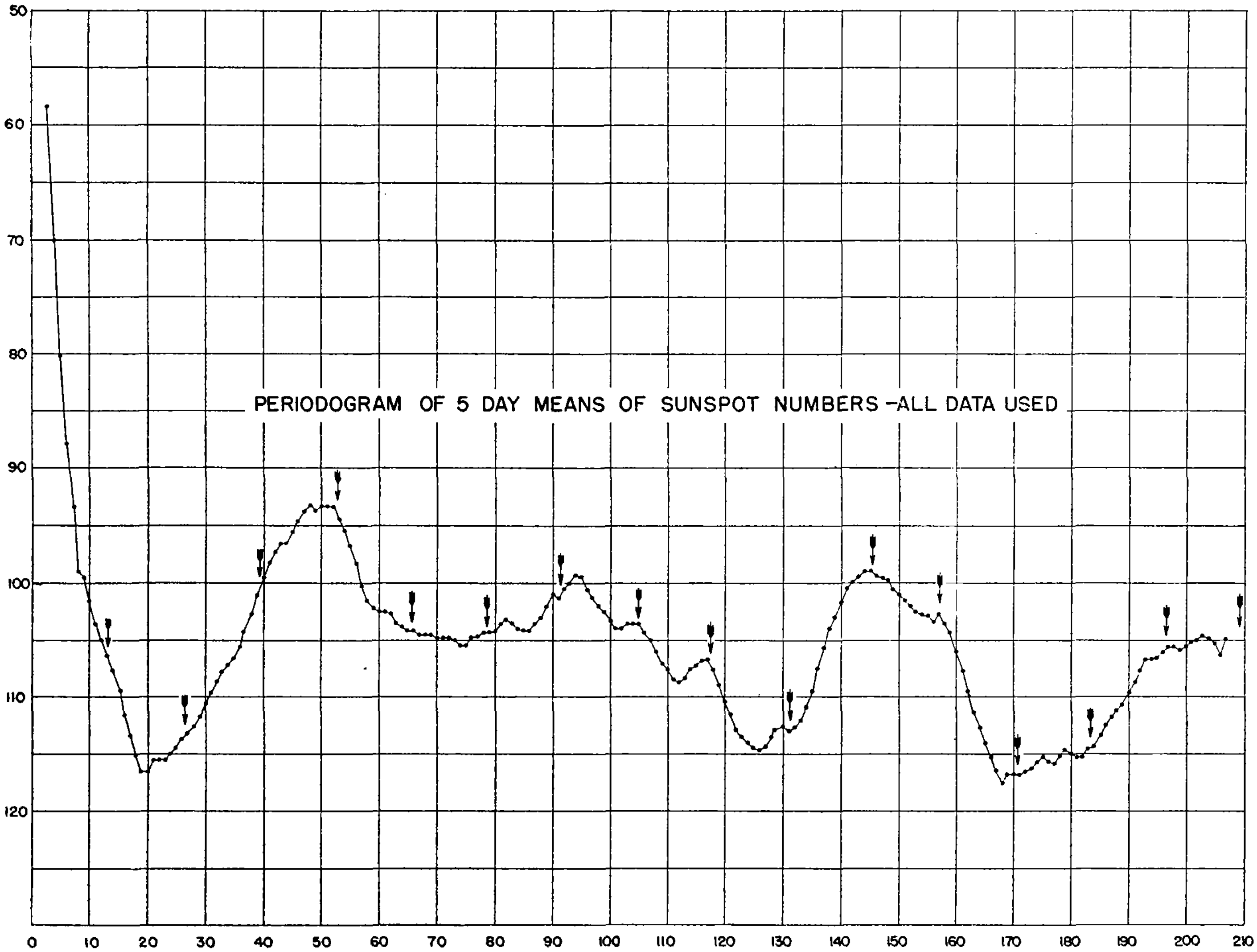

FIGURE 4.

on it. This longer cycle is so nearly equal to 1 year that it seems probable that such is the true length. This does not mean, necessarily, that a cycle of length 1 year existed on the sun (although that is possible), for the result is most probably explained by the annual variation of altitude of the sun in the sky, which would tend to reveal less of the small spots during the winter than during the summer. An objection to this conclusion lies in the fact that this annual term is not found in the other parts of the data.

The short period found is much more regular in its appearance and is of decidedly greater amplitude than that which was indicated for the first 11 years. However, the length of the period is nearly the same as in the former is so excellent, that it seems useless even to calculate probabilities of its reality. This cycle has repeated itself a dozen times in the course of the data used for this third periodogram. Its length is one-eighteenth of the interval between the sunspot minima which limit these data. This is half the length of the only rainfall cycle that has been found very widely on the earth, namely one whose length equals the ninth harmonic of the sunspot cycle. This periodogram shows also the superposition of a short-period term of length 58.8 days.

It should be noted that beginning with 1867.2 four consecutive intervals between sunspot minima are as follows: 11.7, 10.7, 12.1, 12.0 years. The intervals covered by this investigation are the last three of these. The first periodogram covered a cycle which was a year shorter than the preceding one and 1.4 years shorter than the following one. The variation of the cycle indicated an unusual disturbance of the sun. The last third, for which a very regular periodogram pattern has been found, occurs at a time when there has been little change in the length of the cycle and when, apparently, solar conditions have become better stabilized. It would appear reasonable that such conditions would better exhibit such short cycles as exist.

The fourth periodogram is that from the whole 35 years of data. The individual points each represent the combination of approximately 2,500 pairs, with fully 500 independencies. The expected index from random distribution is 10.0. The amplitude of the 250-day cycle is 62 percent of this value in a long enough interval that it has been repeated more than 50 times. The chance of getting such a cycle by pure random fluctuation is only one in billions. We must not, however, jump either at a conclusion that it is constant in length and that it has acted continually, or that the opposite has been true. It well may be that it has been constant or nearly so, despite the negative evidence of the first periodogram but such has not been proved. The shorter cycle shows plainly in the total record and with a length of 65.4 days. This is Dr. Dagobert's period but with greater exactness of description.

The sunspot data beginning with October 1936, and running at least till late in 1937, illustrate almost perfectly why such a cycle as the 250-day one might exist continuously but fail to show in certain epochs.[3] They also can explain an apparent variation in the length of the shorter cycle. During these months the very great majority of the sunspots have occurred on one-half of the sun, so that even with no actual variation there would appear to be a very strong and regular periodicity of length equal to the synodic rotation period of the sunspot belt, i. e., 27.0 days. Such predominance of a hemisphere has been noticed as continuing through many years for northern and southern hemispheres but apparently little search has been made for a longitudinal localization. Figure 5 shows a sine curve of period 250 days, with omission of 10 days' data, at 27-day intervals. This is the type of phenomenon which now is actually present on the sun.

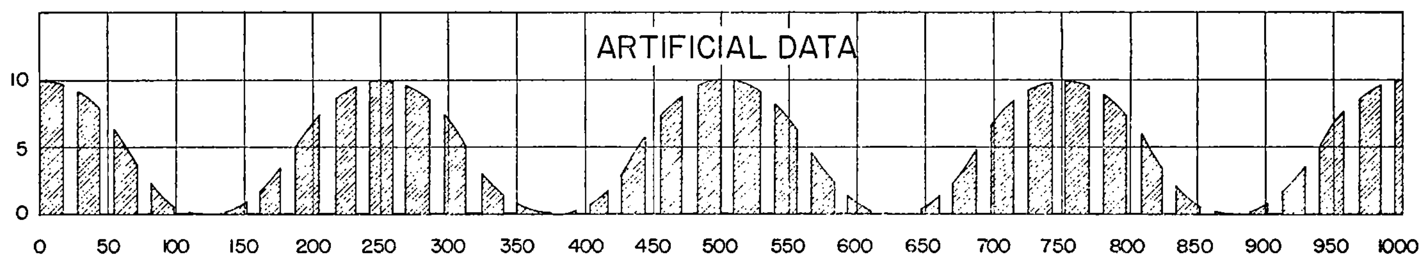

FIGURE 5.

The ordinates have been read from this curve as modified by the missing data. Considering the amplitude of the sine curve as 10, we find the mean of the artificial data to be 3.3. Their standard deviation is 4.1 and the expected random periodogram index is 5.8 For a lag of 250 days the index is found to be 4.5 instead of zero. Had a less perfect curve been assumed, the localization would have masked it even from the periodogram search. Such a cycle cannot be found in the 1936–37 data even if it does exist.

In conclusion: Two short cycles certainly exist in these 35 years of data; the shorter appears to vary in length, the longer to become inoperative at times. We cannot yet, however, be certain of changes in either. It is *possible* that both are of rather constant length and amplitude.

The writer wishes to acknowledge the efficiency of Miss Sylvia Burd, who spend nearly a thousand hours making the necessary routine calculations.

[3] Alter, Dinsmore. 1937. Recent Sun-Spot Peculiarities. Publications of the Astronomical Society of the Pacific. Vol. 49, No. 291, p. 242.